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ON SECULAR ACCELERATION OF PERIODICAL MOTION
OF COMETS

by

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ON SECULAR ACCELERATION OF PERIODICALMOTION OF COMETS.

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A.D. Dubyago

SUMMARY

Comparison is made in this work between secular variations of the mean twenty-four-hour motion and eccentricity, independent of planets' perturbation, for comet Encke according to Asten research, for comet Biela according to Hepperger's research, for comet Brooks according to author's theory, for comet Winnecke according to Haerdtl's works, and for comet Wolf according to Kamienski theory.

It has been proven, that alongside with secular acceleration there exists a secular deceleration of motion, which incidentally does not reach the high values attained by the acceleration.

Hypotheses used for the explanation of these motion anomalies of planets, are questioned. In particular, the results obtained for comet Brooks contradict the Backlund's hypothesis about comet's encounter with meteor streams. Investigation of Bessel's hypothesis showed that anomalies in the motion of comets proved to be the reaction on the masses ejected by the comet's body. If this hypothesis is correct, then we must admit the ejection by the comet, of not only gaseous, but also of hard particles, in agreement with Bredikhin's theory about the role of comets' anomalous tails.

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In my article published several years ago [1], I indicated that the digression from the gravitational theory of the motion of periodical comets is not an exception, but a rule. Actually, this was well known long ago, but seldom stressed in such a general form, probably because the material that could be used for a detailed study of the motion of periodical comets is still too scarce.

In fact, precise calculation of perturbations are available for only few planets, encompassing sufficient time intervals which is naturally explained by the extreme labor-input of such a work. Utilizing the already completed investigations about some well known comets, let us compare here the results that may be derived for our objective.

Turning our attention, first of all to comet Encke, we have to note again the unusual difficulty in the construction of its theory, of

which the classical works of Asten and Backlund are evidence. Precisely for this comet the secular acceleration was discovered with a peculiar prominence shortly after the first Encke's computations later it was established, that even the orbit eccentricity is progressively varying.

Unfortunately, Backlund did not leave any sufficiently reliable definition of eccentricity immediately from observations (he derived this variation according to his own theory through secular acceleration); moreover many significant Backlund's computations have never been published to the present day [2]. Therefore, we have to turn to an older memoir by Asten [3], where computations to our interest are contained.

Further, let us assume in the first approximation that elements of orbits, can undergo linear variation with time and moreover let us introduce the secular acceleration into the mean anomaly, assuming at the same time that it can be expanded in series by powers of time and limiting ourselves to the cubic term. We thus can write the following formulas:

$$\begin{aligned} M &= M_0 + \mu(t-t_0) + \mu'(t-t_0)^2 + \mu''(t-t_0)^3, \\ \varphi &= \varphi_0 + \varphi'(t-t_0), \\ &\dots\dots\dots \end{aligned} \quad (1)$$

From observations of Comet Encke between 1819 and 1865, and taking into account the action of planets from Mercury to Saturn, Asten computed not only the secular acceleration, but also the variation of eccentricity. We should not preoccupy ourselves too much with the criticism of Asten's methods and particularly of his assumed planetary masses; it is well known that his values of Jupiter's and Earth's masses are erroneous, and this undoubtedly should reflect on the accuracy of the quantities searched for. However, there is a question of such large effects, that in the aggregate the relative error is found to be insignificant.

Assuming comet Encke's average period for the indicated time to be equal to 1207.6 days, we obtain, after Asten (page 98) the following values:

$$\mu' = +0.1044184 \pm 0.0001353, \quad \varphi' = -3.680 \pm 0.150$$

which he denotes as variations of corresponding elements for one revolution which in our terms will be:

$$\mu' = +45232'' \cdot 10^{-9} \pm 56'' \cdot 10^{-9}, \quad \varphi' = -3047'' \cdot 10^{-6} \pm 124 \cdot 10^{-6}.$$

Subsequently the secular acceleration of comet Encke, was greatly reduced, and it can be assumed, that its value does not now remain constant over the extent of a few revolutions of the comet.

The movements of Comet Biela was studied by J. Hepperger [4]; it

is true however, that his research was also left somewhat incomplete. He called attention to the presence in this comet of secular acceleration and of eccentricity angle variation.

During the period from 1806 to 1832, when the comet constituted one whole body, i.e. before its partition, he found, by counting in reverse of time:

$$d\mu = -0.144 \cdot 31, \quad d\varphi = +15.99.$$

This period is equal to 9824 twenty-four-hours; therefore in our denotations we shall obtain

$$\mu' = +7345'' \cdot 10^{-9}, \quad \varphi' = -1627'' \cdot 10^{-9}.$$

I have studied the motions of comet Brooks from 1889 to 1940; the results are not yet published. This comet was under observation from 1889-1891, 1896-1897, 1903-1904, 1910, 1925, 1932-1933, 1939-1940 and 1946. In 1922 the comet approached Jupiter by 0.08 a.u., in consequence of which, her orbit underwent great perturbations. I did not succeed in connecting the appearances prior to 1922 with the subsequent ones and it is doubtful whether it will be accomplished with satisfactory precision. It is possible, however, to examine the comet movements separately from 1889 to 1910 and from 1929 up to 1940, which in fact was actually done.

Perturbations were computed from Venus, Earth, Mars, Jupiter and Saturn. For the first four appearances of the comet, it was necessary to introduce into M a term with the cube of time, after which the following was found:

$$\begin{aligned} \mu' &= +5391'' \cdot 10^{-9} \pm 72'' \cdot 10^{-9}, \\ \mu'' &= -1844 \cdot 10^{-13} \pm 75 \cdot 10^{-13}, \\ \varphi' &= -1734 \cdot 10^{-9} \pm 210 \cdot 10^{-9}. \end{aligned} \quad (2)$$

For the occurrences mentioned 14 normal sites were compiled, which are represented by the elements obtained with an average error of one normal coordinate $\pm 1.12''$. Not one deviation reaches $2''$, whereupon let us note, that all the normal sites were assumed with one and the same weight, as it is most rational to do in such cases.

The elements, derived for the appearance of comet Brooks from 1925 to 1940, give for the same quantities:

$$\mu' = +3876'' \cdot 10^{-9} \pm 39'' \cdot 10^{-9}; \quad \varphi' = -1408'' \cdot 10^{-9} \pm 285'' \cdot 10^{-9}.$$

In view of the fact that three appearances were processed, it was not necessary to introduce here μ'' . The average error in the normal coordinate for one of the 16 normal sites, belonging to this cycle of comet's appearance, constituted $\pm 1.52''$. The representations are somewhat inferior to those of the previous case; however, the greater

deviations fall on especially unreliable normal sites.

Let us compare the orbit elements before and after great perturbations in 1922.

EPOCH AND OSCULATION

(1)		(2)	
October 1896 - 11.5 Mean Berlin Time		October 1932 - 3.0 U.T. Time	
M	356°43'	359°4'	
ω°	343.48)	195.50)	
Ω	18.4) 1925.0	177.21) 1925.0	
i	6.4)	5.33)	
ϕ	28.0	29.5	
μ	500.0"	511.2"	

Great changes in ω and Ω are striking. In spite of this, the value of μ corresponds to both cycles of occurrence. In fact, μ' decreases while transiting from elements I to elements II, and the minus sign at μ'' shows the same. Of course, comparison of both values of μ' with the help of μ'' is out of question, as not counting that formula (1) is only a rough approximation of the true independence of time, the determination of μ'' itself rests only on one observation of comet appearance in 1910. Further, the existence of secular changes of eccentricity apparently should not be doubted. It is characteristic that the absolutely independent determination of ϕ for both cycles of comet appearance gave matching values according to their sign, but, as in the case of μ' , the effect was decreased.

Comet Winnecke belongs first of all to the group of well investigated objects. For the five revolutions from 1858 to 1886, during which time the comet was observed in four occurrences, E. Haerdtl [5] conducted an elaborate discussion of observations and a detailed computation of perturbations from all the planets starting from Mercury up to Uranus. He was able to present quite satisfactorily all the occurrences without resorting to any pure variant of gravitational theory. As at that time the mass of perturbing planets were not known with adequate accuracy, and since the perturbation from Jupiter were especially great, Haerdtl introduced into his conditional equations a correction to Jupiter mass as an additional unknown, after which he found

$$m = 1 : (1047.1752 \pm 0.0210).$$

In his reference about Haerdtl's work, E. Schonfeld points out that it would be imprudent to give much credence to the obtained mass, as is done by Haerdtl, who is guided by the average error; Schonfeld says, that the deduction of planet masses from the perturbation, they

produce in comets' movement could be made provided this motion fully agreed with the law of gravity, otherwise the problem could hardly admit a reliable solution. This opinion of Schonfeld should even now, be considered entirely correct, and its best confirmation is the value of Mercury mass, computed at different times from the theory of comet Encke: they oscillate within the broadest limits. The final values of comet Encke's perturbation, obtained by Haerdtl, belong to the following planet masses alongside with which their contemporary values are compiled [6]:

	Haerdtl	New Values
Mercury.....	1:5205000	1:7000000
Venus.....	1:412150	1:406500
Earth.....	1:322883	1:330000
Mars.....	1:3093500	1:3093500
Jupiter.....	1:1047.1752	1:1047.35
Saturn.....	1:3501.6	1:3500
Uranus.....	1:22000	1:22869

The difference in Jupiter's mass should be specially noted. The mass found by Haerdtl, is undoubtedly too great. The last value of Jupiter mass, derived by de Sitter in 1938 is 1:1047.40, i.e. it is still smaller than the Newcom value admitted in [6].

However, it is quite possible to bring the Haerdtl perturbations to the contemporary system of planetary masses. I obtained 12 normal Haerdtl spots for correction to perturbations from all planets (besides Mars, for which even now the same mass is assumed as in Haerdtl). Corrections of element perturbations were transformed into corrections of geocentric coordinates, after which it was revealed, as should have been expected, that the good representation of observations by Haerdtl's theory was entirely due to Jupiter mass admitted by him, while for a contemporary value of the latter there still remain departures from the observed portions of the comet by tens of seconds of arc.

In order to correct the orbit, Haerdtl composed 24 conditional equations. Coefficients for the unknown μ' and ϕ' were additionally computed by me, and then the entire system of equations was resolved anew, which gave the following final elements of Winnecke comet, susceptible to replace those of Haerdtl:

EPOCH AND OSCULATION

Mean Berlin Time - 11.0, March 1875

M_0	359°48'	16.04"	
π	276.41	54.04)
Ω	111.33	38.17) 1880.0
i	11.17	5.72)
ϕ	47.48	59.24	
μ	619.585012"		
μ'	-212·10 ⁻⁹	± 21·10 ⁻⁶	
ϕ'	-193·10 ⁻⁶	± 177·10 ⁻⁹	

The average error of one normal coordinate was obtained equal to $\pm 4.86''$. This quantity is rather great, the departures themselves, particularly those along declinations, were not very satisfactory; but at any rate, the agreement in normal spots are burdened by notable observation errors.

The work of C. Hillebrand [7] about the appearance of Comet Winnecke in 1892, unfortunately was not taken advantage of. In this work substantial deficiencies were found, such as perturbations during the period of comet's visibility were not accounted for although these perturbations, from the Earth in particular, are noticeable enough and then there are plain calculating errors, the influence of which is hard to eliminate. All this was revealed during a more thorough review of Hillebrand's article.

The processing of five appearances of Wolff's periodical comet from 1884 up to 1919, was done by Kamienski. He took into consideration the perturbation of all comets from Venus to Uranus, and compiled fifty normal spots. Here again, the motion does not satisfy the gravitational theory; the digression of normal spots reaches $15''$ and has a sharp systematic character. In the second part of his work, Kamienski [8] computes for each pair of comet's adjacent appearance his own element system and he detects by means of this method the secular variations of elements Ω , π , i , μ (n - in Kamienski's denotations); as to ϕ , no substantial variations were apparent for this element. The quantity denoted by us as μ' , equals according to Kamienski, to $-21 \cdot 10^{-8}''$.

We reach the following conclusions:

1. For none of short periodical comets considered by us does the motion agree with the theory constructed on the basis of the Universal gravitation law.
2. It became known after Kamienski's work and presently solidly established that both the secular acceleration and the secular deceleration exist for comet motion. Although at present hardly anyone takes seriously the Olbers-Encke hypothesis about resisting medium, the very fact of motion deceleration already speaks against it directly, and at the same time also against the explanation of all anomalies in comets' motion by light pressure of the Sun.
3. The secular acceleration of Brooks comet existed from 1889 through 1910, and also from 1925 to 1940, though in 1922 the orbit disposition in space changed strongly.

Backlund assumed that the cause of Encke comet's secular acceleration consisted in comet's collisions with particles of meteor swarm imparting it an additional impulse. But this explanation is hardly valid for the Brooks comet, for after its orbit position changed, it could not, generally speaking, meet with the very same meteor stream.

4. Although the number of our objects is very small, it may be seen from the following comparison that the values of secular acceleration are much greater than the corresponding values of deceleration and this is probably a general rule (for no one has detected to date any motion deceleration in any comet). We have

			μ	ϕ	$10^9 \mu'$	$10^6 \phi'$
Comet	Encke	1819-1865	1073.9"	57°49'	+ 43232"	- 3047"
"	Biela	1806-1832	530.6	48 28	+ 7345	- 1627
"	Brooks	1889-1910	500.0	28 0	+ 5391	- 1734
"	"	1925-1940	511.2	29 5	+ 3876	- 1408
"	Winnecke	1858-1886	619.6	47 42	- 212	- 193 (?)
"	Wolff	1884-1919	520.1	33 49	- 210	(?)

Nothing specific can be said about eccentricity variations for comets Winnecke and Wolf, but for Encke and Brooks comets, and, apparently for the Biela comet, these variations are not subject to doubt: their average error is many times smaller than the obtained values of ϕ' .

An unknown force inducing secular perturbations in μ and ϕ may be conceived as broken down into components in a standard system of mobile coordinates, and namely: into components S along the radius-vector and T perpendicularly to it in the orbit plane (the third component W does not perturb ϕ and μ). The least thought should be given the idea that μ and ϕ vary uniformly; in reality, the obtained values μ' and ϕ' refer by the very point to a full revolution period of comet U and they are given for 24 hours only for the sake of convenience. Applying the well known formula of orbit elements' perturbations, we may write

$$\begin{aligned}\phi' &= \frac{\sqrt{a}}{kU \arccos 1''} \int_0^U \sin v S dt + \frac{\sqrt{a}}{kU \arccos 1''} \int_0^U (\cos v + \cos \varepsilon) T dt \\ \mu' &= -\frac{3 \cos \varphi}{2aU \arccos 1''} \int_0^U \sin v S dt - \frac{3 \cos \varphi}{2U \arccos 1''} \int_0^U \frac{T}{r} dt.\end{aligned}\quad (3)$$

In this last formula, we assume $\mu' = \frac{1}{2} \frac{d\mu}{dt}$, as this follows from (1). We shall make the usual assumption that perturbations take place only in the direct vicinity of the perihelion of comet's orbit, and in accordance with this we shall postulate $\sin v = 1$, $\cos v = \cos \varepsilon = 1$, $r = a(1 - \sin \phi)$. We shall have

$$\begin{aligned}\phi' &= \frac{2\sqrt{a}}{kU \arccos 1''} T \delta t, \\ \mu &= -\frac{3 \cos \varphi}{2a(1 - \sin \varphi) U \arccos 1''} T \delta t = -\frac{3 \cos \left(45^\circ - \frac{1}{2} \varphi\right)}{2aU \arccos 1''} T \delta t,\end{aligned}\quad (4)$$

where δt is the integral of time, during which perturbations act, whence

$$\frac{p'}{\mu'} = -\frac{4}{3} \frac{\lg \left(45^\circ - \frac{1}{2} p \right)}{\mu \sin 1''} \quad (5)$$

It is easy to see that with our assumptions $q = \alpha (1 - \sin \phi)$ remains invariable, since the force T will not affect the position of the perihelion point; hence the last formula is obtained easily for the comets Encke, Biela and Brooks. The ratios ϕ' / μ' , computed with the aid of (4) and found directly by the observed μ' and ϕ' , will be:

			ϕ' / μ' comp.	ϕ' / μ' observed
Comet Encke	1819-1865	74	70
" Biela	1806-1832	262	222
" Brooks	1889-1910	330	322
" "	1925-1940	317	363

The observed and computed quantities coincide within the precision limit of determination of μ' and ϕ' , which was revealed by Asten for the comet Encke. Now this has been confirmed also for the comets Brooks and Biela.

In 1836, Bessel expressed a hypothesis, whereby the secular acceleration of comets is due to the simple case, that every ejection by the comet of a part of its mass, produces a reverse action necessary for the ejection force. That comets eject part of their substance while approaching Sun is a trivial fact, but it is much harder to determine the direction of this ejection relative to radius-vector. It is possible to presume that it does not pass exactly through the Sun, but deviates slightly from the radius-vector to either side: therefore there should exist a component T of the perturbing acceleration with either sign:

Let us assume, that during the time δt action of the perturbing force the comet loses a mass δm ejected with a velocity \underline{v} . If the mass \underline{m} of the comet (which may fundamentally be considered as the mass of the nucleus), we shall obtain for the component in a given case due to reaction forces applied to comet's nucleus.

$$mT\delta t = -v_t \delta m, \quad (6)$$

where v_t is the projection of the ejection velocity \underline{v} on the axis, perpendicular to radius-vector, we consider as positive in the direction of comet's action in accordance with the definition of symbol T . Formula (5) shows that there will be no variation in the total quantity of motion during the ejection by the comet of part of its mass. Hence we shall have:

$$T\delta t = -\frac{v_t \delta m}{m},$$

which could be substituted in (3):

$$\mu' = \frac{3 \lg \left(45^\circ - \frac{1}{2} \varphi \right)}{2aU \operatorname{arc} 1''} \frac{v_t \delta m}{m}.$$

Let us determine the unknown combination of quantities $\frac{v_t \delta m}{m}$:

$$\begin{aligned} \frac{v_t \delta m}{m} &= \frac{2}{3} \mu a U \operatorname{arc} 1'' \lg \left(45^\circ - \frac{1}{2} \varphi \right) = \\ &= \frac{4\pi}{3} \frac{\mu'}{\mu} a \lg \left(45^\circ - \frac{1}{2} \varphi \right) = \frac{4\pi k^{2/3}}{13} \mu' \mu^{-5/3} \lg \left(45^\circ - \frac{1}{2} \varphi \right), \end{aligned}$$

where k is a Gauss' constant in seconds of arc, or, if we consider the velocity, not in A.U. during the 24 hours, but in km/sec (introducing the multiplier 1732).

$$\frac{v_t \delta m}{m} = 1.688 \cdot 10^6 \mu' \mu^{-5/3} \lg \left(45^\circ - \frac{1}{2} \varphi \right). \quad (7)$$

Substituting here the corresponding quantities, we have, for our investigated comets:

	$10^6 \frac{v_t \delta m}{m}$	$10^5 \frac{v_t \Delta m}{m}$	No. of revolutions
Comet Encke	187	262	14
" Biela	135	54	4
" Brooks 1889-1910	174	52	3
" " 1925-1940	118	47	4
" Winnecke	3.1	1.5	5
" Wolf	5.6	2.8	5

Two revolutions of 1910-1925 were also taken into account for the Brooks' comet and, according to still unpublished computations by the author, the value μ' corresponding to the second period of observations, was taken for them.

If we take for v_t a quantity somewhat lower than the thermal velocities of gas molecules, for example, approximately 0.2 km/sec, the comet mass losses reach magnitudes from 0.3% (Biela comet) to 1% (Encke comet) of the total nucleus mass. Thus, to consider that ejected are only the gases desorbed by comet nucleus' lumps, is impossible, for the entire reserve of these gases could not reach such a great fraction of comet's total mass.

According to B.A. Vorontsov-Velyaminov [9] investigations of comet's mass and the structure of comet nucleus, the logarithm of the

number of ionized molecules C_2^+ , being lost by comet Gallea during one revolution, is:

$$\log M^+ = 35.23,$$

whereupon the loss of these molecules plays a dominant role in the total gas loss. Multiplying by the molecular weight C_2 , equal to 24, and by proton's mass $1.66 \cdot 10^{-24}$ g, we obtain the loss of comet's mass $\delta m = 6.8 \cdot 10^{12}$ g, which should be compared with the total mass m of this comet's nucleus, which Vorontsov-Vel'yaminov assumes to be equal to $3 \cdot 10^{19}$ g. This gives:

$$10^6 \frac{\delta m}{m} = 0.23$$

None of the admissions in respect to velocity v_t not contradictory to common sense, nor the rather substantial uncertainty of Vorontsov-Vel'yaminov's estimates can bring this value into an agreement with our determinations $v_{t, \frac{\delta m}{m}}$, giving numbers many times greater.

We conclude, that if Bessel's hypothesis about the reason of secular acceleration is generally correct, then the question should not be of the gaseous-like ejections, but of the hard particles of comet's nucleus. In this respect one may recall the idea, more than once advanced by F.A. Bredikhin, that anomalous comet tails are evidence of the formation process from a comet of meteor series in the ejection process of hard particles, that has become visible.

To ascertain how such an ejection can take place and whether or not, as Bredikhin supposed, the comet-ejected gases could carry along the hard particles in sufficient number and with adequate velocity, and in particular, where the necessary energy for this comes from, is a question of future. It is possible, that such a research can also throw a ray of light on the nature of secular accelerations of comets which until now is still very vague.

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*** THE END ***

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